Measurements in Freejet Mixing/Combustion Flows

LEONARD S. COHEN* AND ROY N. GUILE†
United Aircraft Research Laboratories, East Hartford, Conn.

The effect of combustion on the mixing of axisymmetric, supersonic, turbulent free jets was investigated experimentally using cooled probes, an Ebert spectrophotometer, high-speed motion pictures, and schlieren and ultraviolet photographs. Results of this work, including detailed Pitot pressure, temperature, and species concentration profiles are presented for the mixing of a Mach 1.46 central jet of cold hydrogen with a vitiated Mach 1.86 concentric outer jet. Flow conditions in the vitiated jet were adjusted to provide two different values of oxygen concentration in an effort to identify the separate effects of mixing vis-a-vis combustion. Static temperatures of the vitiated streams exceeded the H_2/O_2 autoignition value of 1000°K. Pressures in both jets were matched at approximately 0.9-atm. Samples aspirated from the flow through quenching probes were analyzed for species concentrations by employing an on-line gas chromatograph. Water concentration and local temperature were also measured spectrophotometrically. A comparison of mixing rates in equivalent combusting and inert jet flows showed little difference for the conditions studied. The use of a kinematic eddy viscosity model which includes the effect of preturbulence and the variation of density through the mixing layer yields analytical results which are in agreement with the test measurements.

Nomenclature

a = spectral absorptivity
b = transverse extent of mixing layer, m
c = sound speed, m/sec
ER = equivalence ratio, $kgH_2/(kgH_2)$ stoichiometric
f_{H_2} = fraction of hydrogen unreacted
F = correlating parameter or "history," see Fig. 12
F' = history adjusted for intermixing effect, see Eq. (4)
I_e = gas spectral emission intensity
k = length of enclosure, m
m = ratio of vitiated stream velocity to centerline velocity
m_1 = value of m at which preturbulence mechanism becomes important
M_j = mass fraction of species j
$\Delta M_j = M_{j℄} - M_{ja}$
n = ratio of vitiated stream density to centerline density
n_1 = value of n when preturbulence mechanism becomes important
P = pressure, atm
$r_{1/2}$ = half-thickness of mixing layer defined using Eq. (10)
T = temperature, °K.
u = longitudinal velocity, m/sec
W = mass flow rate, kg/sec
x = longitudinal distance from nozzle exit plane, cm
x_1 = axial station at which preturbulence mechanism becomes important, cm
z = transverse distance from centerline, cm
β = flame half-angle
ϵ = kinematic eddy viscosity, m^2/sec
θ = time, sec
κ = universal constant in mixing models
λ = wavelength, m
ν = frequency, sec^{-1}
ρ = density, kg/m^3
τ_{ID} = ignition delay time, sec
$\Delta\tau$ = residence time, sec

Subscripts

a = vitiated stream
℄ = centerline
i = hydrogen jet
j = species designation
k = segment designation
n = radial zone
o = core
q = streamline designation
r = resonance
t = stagnation condition

I. Introduction

WIDESPREAD interest in advanced airbreathing propulsion devices has motivated numerous analytical studies of the mixing and combustion of compressible, turbulent streams. These analyses are basically either of the integral type[1,2] or the detailed computational type in which the pertinent differential equations are solved directly along streamlines using numerical techniques.[3-5] In each case, some empiricism must be introduced in order to specify the rate of mixing of fuel and air streams. The validity of these analyses is limited therefore by the extent and accuracy of available information on jet mixing rates and/or kinematic eddy viscosities. Since suitable flow facilities and instrumentation for use in supersonic combusting studies have only recently become available, these analyses have not, in general, been subjected to experimental confirmation.

The present objective was to obtain detailed data, including Pitot pressures, temperatures, and species concentrations throughout constant pressure mixing and combusting jet flowfields. This data was to be sufficiently definitive to reflect any significant influence of chemical heat release on mixing rates, and to allow the characterization of mixing rates with a formulation for the kinematic eddy viscosity. In addition, the techniques of on-line gas chromatographic analysis and spectrophotometric determination of temperature and water concentration in axisymmetric flows were to be evaluated for use in combustion test environments.

II. Experimental Study

Discussion

An investigation of the effect of combustion on mixing requires the collection of and comparison between comparable measurements made in equivalent inert and reacting jet flows. The coaxial freejet system chosen to produce such equivalent flows consists of a central hydrogen jet surrounded by a high-

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* Supervisor, Aerothermochemistry Group, Air-Breathing Propulsion Section. Member AIAA.

† Associate Research Engineer, Aerothermochemistry Group, Air-Breathing Propulsion Section. Member AIAA.

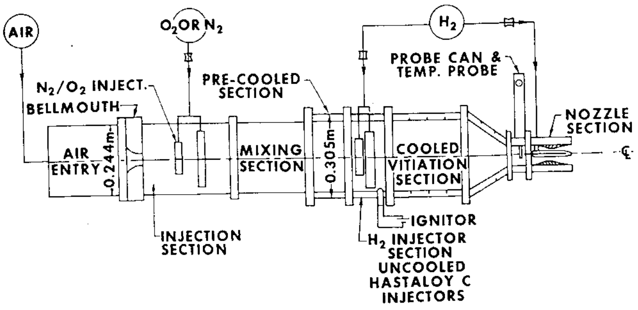


Fig. 1 Schematic diagram of test facility.

temperature annular jet containing oxygen, nitrogen, and water; a less reactive flow is produced by replacing most of the oxygen in the annular stream with nitrogen.

A high temperature is required in the annular jet to insure the rapid autoignition of hydrogen in the mixing layer formed between the jets. Heating of the annular jet is accomplished by vitiation. Hydrogen and air are combusted upstream in a vitiation chamber and additional makeup oxygen and/or nitrogen is added to obtain the desired weight flow and oxygen concentration. Vitiation was selected over alternative heating methods, including the use of a pebble bed,[6] arc jet,[7] shock tube,[8] or rocket[9] since associated run times can be relatively long, depending on fuel and makeup gas storage capacity. Run times achieved ranged from 30 to 50 min. As a result of vitiation, water, and the intermediate species H, O, and OH will be present in the annular jet. The influence of these contaminants on the reaction kinetics in the freejet region must be accounted for in any mixing and combustion analysis.

Test Apparatus

Experiments were conducted using the water-cooled test rig in Fig. 1. High-pressure air is passed through a bellmouth flowmeter into the injection section where it is mixed with either makeup oxygen or nitrogen injected through flat-face rings. The resulting flow is then mixed and burned with hydrogen in the vitiation section. In tests requiring a reactive flow comparable to air, i.e., vitiated air, sufficient oxygen is injected so that it comprises approximately 21 vol. % of the resulting vitiated stream. The total temperature of the vitiated air stream is typically 1750°K. The same temperature may be obtained for an inert (nitrogen) vitiated flow by reacting the oxygen in the air with hydrogen, and making up the desired total weight flow with additional nitrogen injection. In either case, the resulting vitiated flow is passed successively through a conical reducing section and an annular converging-diverging nozzle. The inner surface of the nozzle is formed by a centerbody held in place by three equispaced oval struts (see Fig. 2); struts and centerbody are coated with 0.41-mm of zirconia insulating material. Hydrogen introduced into the centerbody via the support struts is channeled into a collection chamber and directed onto the aft end of the centerbody to provide impingement cooling. The H_2 flow is finally

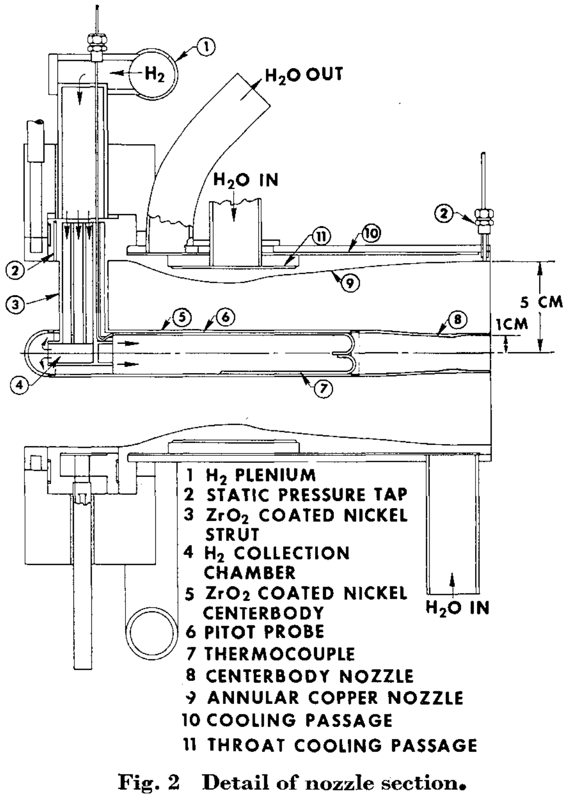


Fig. 2 Detail of nozzle section.

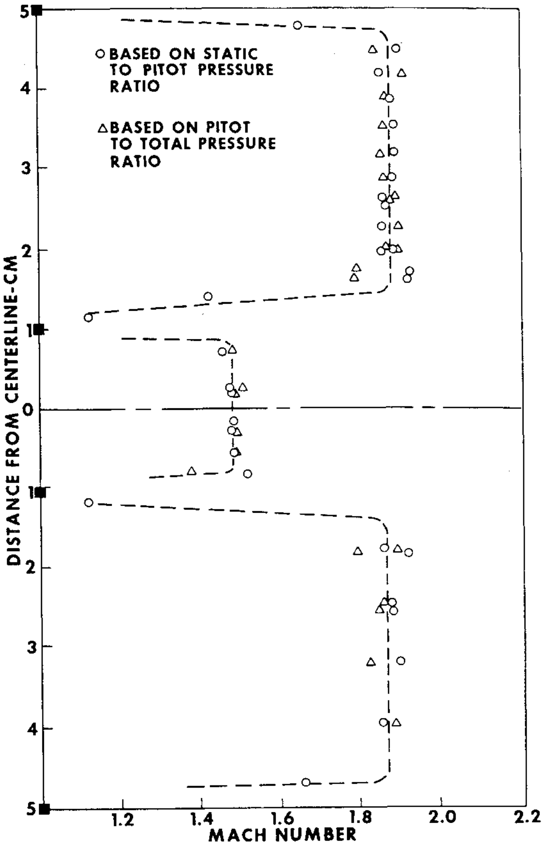


Fig. 3 Mach number profile at nozzle exit plane.

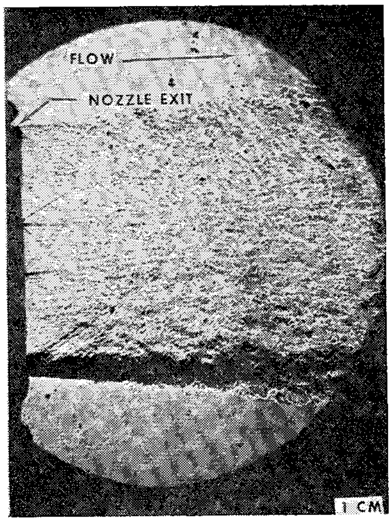


Fig. 4 Schlieren photograph of jet flow at nozzle exit plane.

brought into contact with the vitiated stream at the nozzle exit to mix (and burn) in the following freejet test region.

The copper annular nozzle was designed for uniform, parallel, shock-free flow. Tests conducted with the nozzle installed at the rig to verify the design yielded the Mach number distribution shown in Fig. 3. The edge of the centerbody and annular nozzle are located by bars on the ordinate of the figure. As seen from Fig. 3, the annular flow is uniform at a Mach number of 1.86 and a constant Mach number of 1.46 is characteristic of the centerbody flow. Mach numbers computed from both the Pitot-to-total pressure ratio and the static-to-Pitot pressure ratio are identical, thereby confirming that the flow leaving the nozzle is shock-free. A schlieren photograph taken at the nozzle exit plane is shown in Fig. 4.

Boundary-layer thicknesses at the nozzle exit plane on the outer and inner centerbody surfaces were calculated to be 1.85-mm and 1.15-mm, respectively. These boundary layers and the 0.54-mm centerbody lip constitute a 3.54-mm-thick ring which appears as a wake in the freejet flow. Velocities in a wake are lower than in the surrounding fluid so that protracted fluid residence times and reduced ignition lengths result. However, mixing, the very process which brings fuel and high-temperature air together, also rapidly increases wake velocities. If wake velocities are increased significantly before proper concentrations of fuel and oxident can be brought together or before temperatures in the wake are raised to 1000°K, then the effect of the wake on ignition will be minimal. From computations made with the mixing and combustion analysis (described in Sec. IV), this appears to be the case for the present experiments.

Instrumentation

Pitot-Concentration Probe Rake: A traversing rake containing two Pitot probes and two gas-sampling (quenching) probes, Fig. 5, was constructed for this investigation. Cooling water is supplied to each probe at a rate of 0.12-kg/sec and a pressure of 13.5-atm. After a circuit through the probe, the water is dumped into the low-pressure strut cooling water.

The internal design of the quenching probe was achieved through a one-dimensional study of the governing geometric and flow parameters for the most severe environmental conditions expected. Area variations, wall friction, and heat transfer were all considered in the analysis through the use of influence coefficients.[10] It was desired in the design of the probe to swallow the probe bow shock, inhibit chemical reaction of gas samples injected into the probe by the combined quenching effects of aerodynamic expansion and convective cooling, prevent choking of the flow in the probe and delay shocking down of the flow in the probe until $T_t < 1000°K$. The resulting probe in Fig. 6 has a length of 89-mm and an inlet hole diameter of 0.61-mm. The gas sample flows through a divergent conical nozzle having a 5° half-angle and thence through a 1.88-mm-diam duct. Also given in Fig. 6 is the ignition parameter defined as the ratio of gas residence time to chemical ignition delay time.[4] Note that a value of unity signifies ignition. For a gas sample initially at Mach 1.50, $T_t = 2220°K$, and $ER = 1$, no ignition occurs within the probe, at least on a one-dimensional basis. The possibility of chemical reaction in the wall boundary layer of the sample tube cannot, however, be discounted on the basis of this analysis. At the aft end of the probe, the ingested gas sample must shock down. Nevertheless, chemical reaction will not occur since the computed total temperature of the flow is only 450°K. The gas flow Mach number is maintained near a value of 2.20 reflecting tradeoff between friction and cooling effects.

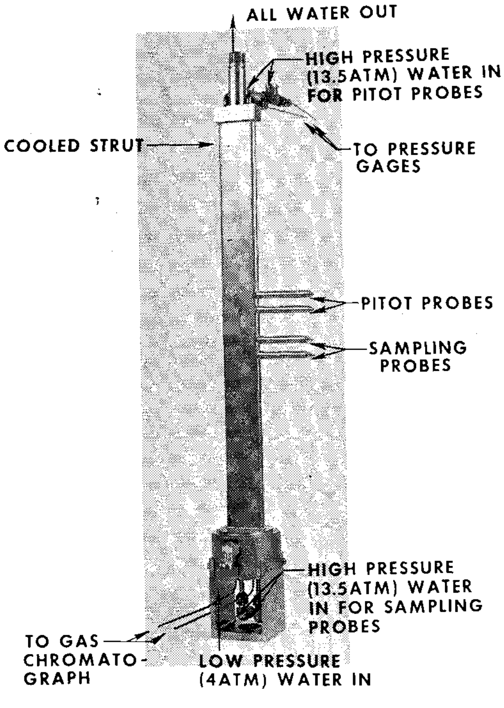


Fig. 5 Pitot-concentration probe rake.

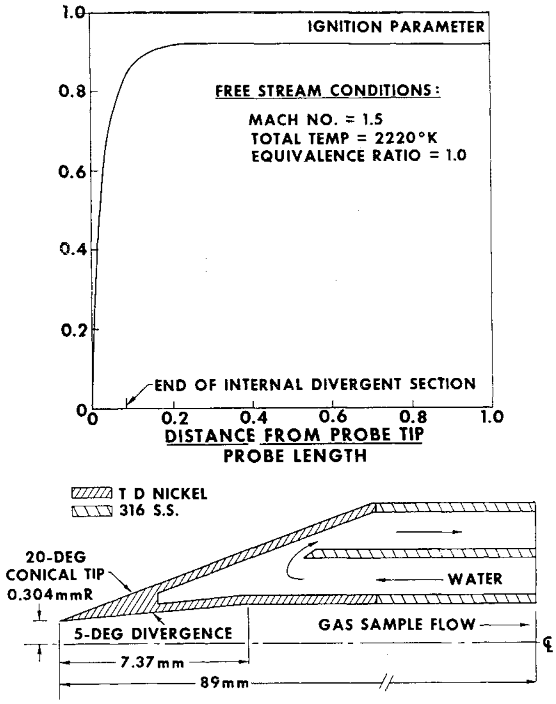


Fig. 6 Schematic diagram of probe with calculations for the ignition parameter.

Gas Chromatograph: During a gas sampling test, material is continuously aspirated from the freejet mixing region into the probes. After a 15 sec purge a portion of the gas is injected into a gas chromatograph (GC) for on-line determination of the concentrations of H_2, O_2, H_2O, and N_2. Lines from the probe to the GC are heated to prevent condensation of water, and the entire GC unit is enclosed in an oven heated to 410°K. The GC system consists of a (91.5% He-8.5% H) carrier gas flow system, a sample injection valve, a separation column, a temperature control system, a detector, and a read-out device. Actually, two separation stages are used since no one column material was found that could separate the four gaseous constituents rapidly. Poropak R (50–80 mesh) in Column 1 separates H_2, O_2, and N_2 from H_2O, and 40–50 Mesh Molecular Sieve 13X in Column 2, effects a separation between H_2, O_2, and N_2 after a drying step. Hot-wire thermal conductivity cells are employed as detectors. A complete analysis including purge is completed in about 75 sec.

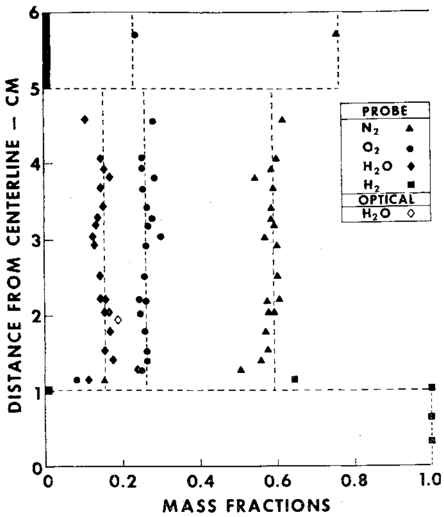


Fig. 7 Initial species concentration profiles—vitiated air condition.

Aspirating Temperature Probes: Total temperatures in the inert freejet flowfield are measured by employing a water-cooled platinum-tipped aspirating probe. The probe contains a platinum, platinum-10% rhodium thermocouple whose junction is situated within two concentric radiation shields. A sample from the jet flow is continuously aspirated through a 4.76-mm hole into the probe, conducted past the thermocouple junction, and exhausted through the water-cooled support member. This probe design yields data which requires no correction for radiation losses.[11]

Ebert Spectrophotometer: A spectrophotometer system[12] is used to obtain local temperatures and H_2O concentrations from infrared emission and absorption measurements. It consists of a Nernst glower light-source, an optical collection system, a wave length scanning monochromator, a lead sulfide detector, and photometric circuitry. Calibration is conducted using a black-body source. Dry nitrogen purging of the entire optical system is provided to eliminate atmospheric water vapor and carbon dioxide absorption. Two additional features of the system are that 1) the entire system can traverse as a unit over a 0.3-m horizontal path, and 2) an arrangement of mirrors produces a collimated beam, permitting any desired working space. The beam cross section is approximately a rectangle 6.34-mm × 3.17-mm with the longer sides normal to the flow direction.

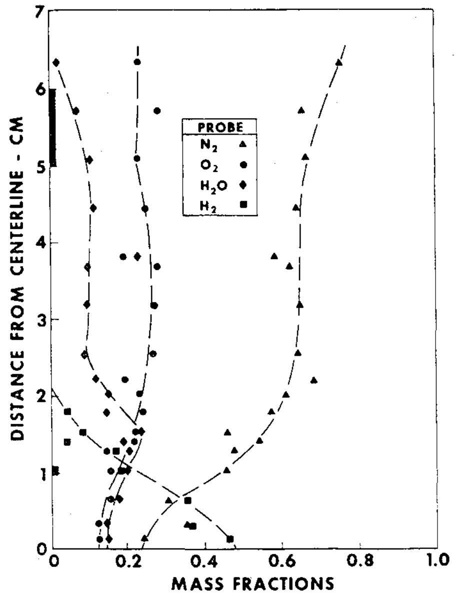


Fig. 8 Species concentration profiles at x = 35.6 cm—vitiated air condition.

Table 1 Supply gas flow rates—kg/sec

Gas	Vitiated air	Vitiated N_2
Air	2.059	2.076
N_2 or O_2	0.540	0.428
H_2 for vitiation	0.045	0.053
Totals	2.644	2.557

Spectrophotometer measurements include the gas spectral emission intensity, $Ie(\lambda)$, and the spectral absorptivity, $a(\lambda)$ obtained at the CO_2-free ν_3 line, i.e., at $\lambda^{-1} = 3837.85$ cm^{-1}, the strongest emitter in the 2.7-μ region. These data from the axisymmetric flame are reduced by an Able integral computer program[13] which corrects for self-absorption to yield the radial-zone quantities Ie_n and a_n. The program also computes the zonal Planck-Kirchoff temperature, i.e., the translational temperature at local thermodynamic equilibrium, following Plass[14] and Tourin.[15] Water partial pressures are computed from the expression derived in Ref. 14 for a thick line and a constant slit function.

III. Experimental Results

Test Conditions

Calibration tests were conducted to determine a set of equivalent vitiated flows, i.e., a reactive air flow and an "inert" nitrogen flow. These tests were necessary since it was not possible to achieve 100% combustion efficiency in the vitiation chamber. During these tests, gas flow rates were varied until the total temperature profiles at the nozzles exit plane were essentially the same for both the vitiated nitrogen and vitiated air test conditions. Supply gas flow rates for the equivalent flows appear in Table 1; the nominal total temperature for these flows was 1750°K. Note that a small concentration of oxygen had to be tolerated in the vitiated N_2 flow. The vitiated streams exhausted from the annular nozzle at a static pressure of 0.9-atm for the conditions of Table 1. This slight under-expansion had to be accepted in order to obtain gas flow rates which were consistent with the capabilities of the gas storage facilities to provide realistic run times. The very weak wave which results from the mismatch with cell pressure has an associated angle which is only 1° larger than the Mach wave angle corresponding to a perfect match of the flowfield pressure.

High-Speed Motion Pictures

High-speed (128 fps) color motion pictures taken at the vitiated air conditions revealed an unsteady but cyclical behavior of the flame in the freejet flowfield. During a typical 0.024 sec cycle, the brightly burning flame disappeared and then reappeared, ultimately regaining its initial luminosity. It was found that sinusoidal pressure fluctuations produced in the cavity of the experimental rig downstream of the air bellmouth were responsible for the instability. These fluctuations have a frequency of about 42 cps and an amplitude of 0.136-atm. In order to maintain a constant weight flow through the rig, therefore, the combustion temperature must also fluctuate. The variation required is ±83°K in the total temperature or ±56°K in the 1140°K static temperature. The effect of such temperature variations on ignition characteristics are profound at temperatures close to the autoignition limit. Thus the changing luminosity of the flame, and even the apparent extinguishment during each flame cycle, are not surprising.

These findings suggest that the pressure fluctuations are associated with vibrations of the gas mass within the test rig

Table 2 Comparison of H_2O molar concentration measurements—vitiated air

x (cm)	z (cm)	Probe	Optical
0.33	1.90	0.217	0.278
5.1	1.90	0.200	0.236
10.2	1.90	0.261	0.284
10.2	4.45	0.237	0.232
17.8	1.90	0.250	0.327
17.8	4.45	0.208	0.265
25.4	1.90	0.163	0.233
25.4	4.45	0.198	0.217

downstream of the bellmouth. While exact calculation of the resonant frequency of the test rig is made difficult by uncertainties in end corrections and the inhomogeneous nature of the gas, a crude estimate can be made from the expression[16]

$$\nu_r = c/4k \tag{1}$$

In Eq. (1), c is the sound speed and k is the length of the enclosure. The sound speed of the combustion gases is approximately 800-m/sec and that of the gas approaching the vitiation chamber is 330-m/sec. Using a length-average sound speed of 610-m/sec, ν_r is calculated to be 46 cps, or roughly the value observed. The time to make any given measurement generally encompasses several cycles of flame fluctuations so that some averaged effect is manifested. It is expected that the "averaged" conditions measured show a greater influence of combustion than would be found in a comparable steady flame situation. This conclusion is based on the observation that ignition times vary exponentially with temperature.[4]

Photographs

UV and visible photographs of the combusting freejet both displayed very shallow flame angles, and ignition lengths less than 3 cm. Flame half-angles measured from the photographs were 2–3°.

Species Concentration Profiles

The sharply defined concentration distributions at the nozzle exit plane in Fig. 7 for the vitiated air conditions are changed through mixing into the smoothly varying profiles of Fig. 8. By the x = 35.6-cm location, significant nitrogen, oxygen, and water from the vitiated stream have penetrated through the central hydrogen stream to the jet centerline. Some spreading of hydrogen beyond the centerbody lip location is also clearly indicated. The presence of a substantial water "bump" verifies that combustion has occurred. Profiles measured at x = 35.6-cm for the vitiated nitrogen conditions are similar to those for the air case. The water bump is still observed but it is less prominent. However, the centerline hydrogen concentration is about the same. In fact, the variation of the centerline hydrogen and nitrogen concentration with downstream distance (Fig. 9) is identical for both conditions when a proper normalization is used to reflect different initial concentrations. Thus any influence of combustion on mixing rates which may be present in the present experiments does not appear to be dramatic.

Water concentrations from spectrophotometer measurements are compared to the corresponding probe measurements in Table 2. The spectrophotometer concentrations are generally larger, but the same trends are indicated in both sets of data.

Temperature Profiles

The total temperature profile at x = 0.33 cm for the vitiated nitrogen test condition in Fig. 10 displays a fairly uniform temperature of about 1700°K. As mixing proceeds, the cold ambient stream and the cold central hydrogen jet penetrate into the vitiated stream to reduce the temperatures at the profile extremities. A temperature peak of 1840°K which occurs in the mixing layer between the vitiated flow and the hydrogen stream results from some combustion of the oxygen carried in the vitiated nitrogen stream. Temperature measurements could not be made for the vitiated air conditions with the available probes. Also shown in Fig. 10 are temperatures calculated from spectrophotometer data. The over-all quantitative agreement between the two types of measurements is only fair with the spectrophotometer-determined values averaging 200–300°K less than the probe values at x = 0.33 cm and 500°K less at x = 25.4 cm. The errors noted above in the temperatures and concentrations which were determined spectrophotometrically, are consistent with measured absorptivities which are too high. It should be noted that although complete scans were taken with the spectrophotometer, a portion of the data was lost due to equipment malfunction.

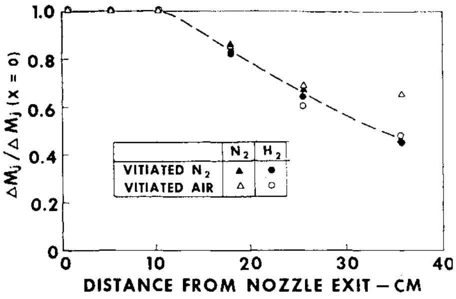


Fig. 9 Centerline concentration decay.

Pitot Pressure Profiles

A comparison between Pitot profiles for the vitiated air and N_2 conditions at x = 35.6 cm in Fig. 11 discloses small differences which are confined to a 2.5-cm radius circular region about the jet centerline. It is not clear from this data whether the differences between the profiles result from combustion-augmented mixing or some other effect such as local flow expansion associated with chemical heat release.

IV. Analytical Study

Mixing and Combustion Analysis

A study of the simultaneous turbulent mixing and chemical reaction of constant pressure, axially directed, supersonic gas streams was conducted using a computer program[4] which provides local conditions along streamlines from a numerical solution of the boundary layer form of the conservation equa-

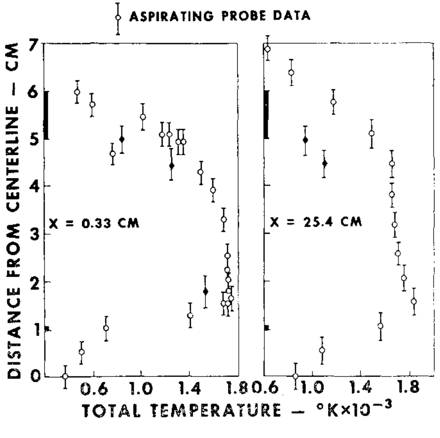


Fig. 10 Total temperature profiles—vitiated nitrogen conditions.

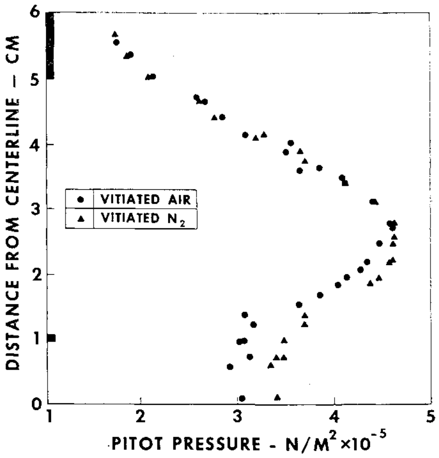


Fig. 11 Comparison of Pitot profiles at $x = 35.6$ cm.

tions in the von Mises coordinate system. Nonequilibrium chemical reaction effects are included by employing a correlation of the ignition and reaction time-history of the H_2/Air/H_2O system. The mixing rate is specified by using a formulation for the kinematic eddy viscosity.

In the analysis, mixing and combustion between streams is treated by dividing the flow into a large number of segments, and solving the conservation equations in the manner of an initial value problem. Within each segment mixing is assumed to proceed independently of any chemical reaction effects. Such effects are subsequently introduced on the basis of (mixed) average conditions within the segment and adjustments are made to reflect heat release and species chemical conversions before proceeding downstream. In this procedure it is assumed that the gases obey the perfect gas law and the flow is shock-free. The turbulent Lewis and Prandtl numbers are taken equal to unity.

The use of the boundary-layer equations to describe mixing and combusting flows has been criticized.[17] Certainly, the assumption of constant static pressure throughout a combusting flow is questionable since mass conservation considerations require that the flow turn away from the centerline as the flame front is approached. In the present experiments, however, the flame angle and hence the deflection angle of the flow upstream of the flame are small, and the use of a constant-pressure turning approximation is reasonable. Another point stressed in Ref. 17 is the difficulty of defining initial conditions for use in the analysis. Fortunately, this is not a great problem when the jets under study are supersonic.

Chemical Model

A correlation for the unreacted fuel fraction, f_{H_2}, as a function of local conditions and the time to traverse the segment, i.e., the residence time $\Delta\tau \equiv \Delta x/\bar{u}$, must be provided for in the analysis. This "history" of the combustion of H_2 was obtained by employing the reaction mechanism and reaction rates set forth in Table 3 in a one-dimensional, constant pressure kinetics computer program. References for the nine reaction rates are 18–26, respectively. Initial conditions for the computer computations, including temperature and concentration of all important species (i.e., H, OH, O, H_2O, N_2, and O_2) were taken from vitiation-nozzle exhaust equilibrium calculations.[27] Time histories were correlated in the form shown in Fig. 12, giving the fraction of unburned hydrogen as a function of time, θ, temperature, T, and equivalence ratio, ER.

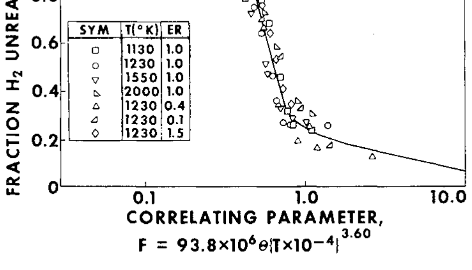


Fig. 12 Correlation of H_2/O_2 chemistry reaction histories.

Table 3 H_2/O_2 reaction mechanism

Reaction	Reaction rates[a] cm³, mole, sec, °K, cal K_1	K_2
$H_2 + O_2 \rightarrow 2OH$	2.5×10^{12}	19650
$H + O_2 \rightarrow OH + O$	2.2×10^{14}	8310
$O + H_2 \rightarrow OH + H$	4.0×10^{13}	5140
$H + H_2O \rightarrow OH + H_2$	1.0×10^{14}	10200
$O + H_2O \rightarrow 2OH$	8.4×10^{13}	9120
$H + O_2 + D^b \rightarrow HO_2 + D$	8.6×10^{14}	−645
$H + OH + D \rightarrow H_2O + D$	$6.0 \times 10^{18}/T$	
$H + H + D \rightarrow H_2 + D$	$3.0 \times 10^{18}/T$	
$O + O + D \rightarrow O_2 + D$	$3.0 \times 10^{17}/T$	

[a] Rates for the first six reactions of the form: $K_1 \exp(-K_2/T)$.
[b] D is inert third body.

The chemical reaction history of Fig. 12 contains an (initial) isothermal preignition period of length τ_{ID}, defined as the time required for f_{H_2} to fall noticeably below unity, which may be located at the value $F = 0.076$. Thus

$$\tau_{ID} = 8.1 \times 10^{-10}(T \times 10^{-4})^{-3.60} \tag{2}$$

is used with an ignition criterion to describe the chemistry in the preignition period. Ignition is said to occur on any streamline q bounding a particular segment of the flow at the axial location x_k where the following condition involving the ignition parameter is satisfied:

$$\sum_{x=0}^{x=x_k} \left(\frac{\Delta\tau}{\tau_{ID}}\right)_{x_k} \geq 1.0 \tag{3}$$

If ignition has already occurred on streamline q then the de-

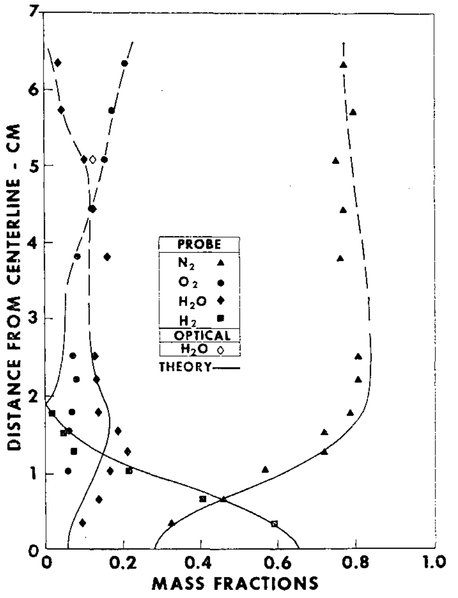


Fig. 13 Predicted concentration profiles at $x = 25.4$ cm compared with data—vitiated nitrogen condition.

gree of chemical reaction is determined from the correlating curve of Fig. 12.

The intermixing of packets of fluid with different reaction time histories is accounted for through an adjustment of the "history," F, for the fluid between streamlines q and $q + 1$. The adjusted value F' associated with a flow of fluid W is obtained from the mass-averaged expression,

$$F' = F\left[1 - \frac{\Delta W_q + \Delta W_{q+1}}{W}\right] + F_q \frac{\Delta W_q}{W} + F_{q+1} \frac{\Delta W_{q+1}}{W} \tag{4}$$

in which ΔW_q and ΔW_{q+1} are the mass flows of fluid packets with histories F_q and F_{q+1}, respectively. The gross quantity of mass transferred $(\Delta W_q + \Delta W_{q+1})$ is[28]

$$\frac{\Delta W_q + \Delta W_{q+1}}{W} = \frac{\Delta x \tan\beta}{(Z_{q+1} - Z_q)} \times \left[\frac{Z_q}{Z_{q+1/2}}\left(\frac{\rho_q}{\rho}\right) + \frac{Z_{q+1}}{Z_{q+1/2}}\left(\frac{\rho_{q+1}}{\rho}\right)\right] \tag{5}$$

where β is the half-angle of the flame front and $z_{q+1/2}$ is $\frac{1}{2}(z_q + z_{q+1})$.

Mixing Model

The computerized analysis can be used to generate a detailed flowfield for given initial and boundary conditions, once a kinematic eddy viscosity distribution has been specified. For this, it is necessary to consider both the turbulence initially present in the jets, i.e., the "preturbulence," and the turbulence resulting from the interactions between the jets. When the velocity ratio $m \equiv u_a/u_{℄} \ll 1$, the growth of the mixing layer is controlled by jet interaction since shearing stresses of large magnitude occur which induce high intensity turbulent activity. As $m \to 1$, however, the importance of the preturbulence contribution, increases. Thus the flow displays a "memory," in that mixing far downstream is controlled by an initial turbulence level. To a significant degree this level is determined by the experimental apparatus so that the value of $m(=m_1)$ at which preturbulence becomes important is a characteristic of the system under consideration. Values of m_1 between 0.4 and 0.6 are found to be representative of a major portion of the existing mixing data,[29] with the larger value corresponding to a low initial turbulence level most appropriate for supersonic jets.

Classically, the constant exchange coefficient hypothesis of Prandtl has been used for the turbulence contribution from jet interaction. This formulation may be written as

$$\epsilon = \kappa(1 - m)bu_{℄} \tag{6}$$

where κ is a universal constant in each region of the jet for a given geometry, and b is the transverse extent of the mixing layer. An extension of this expression to account for density variations through the mixing layer was derived in the form[8]

$$\epsilon = \kappa\left(\frac{1 + n}{2}\right)^{0.8}(1 - m)bu_{℄} \tag{7}$$

The parameter n is the ratio of densities, $\rho_a/\rho_{℄}$.

At some longitudinal distance $x = x_1$ the preturbulence mechanism becomes dominant and a different eddy viscosity model must be employed. This problem was treated in Ref. 30 with the result for the region of the flow downstream of x_1:

$$\epsilon = \kappa\left(\frac{1 + n_1}{2}\right)^{0.8}(1 - m_1)bu_{℄}\left[\left(\frac{1 + n_1}{1 + n}\right)\left(\frac{1 + mn}{1 + m_1 n_1}\right)\right] \tag{8}$$

Derivation of Eq. (8) depends on the assumption that the shear flow induced by the preturbulence is self-preserving, i.e., the distribution of the nondimensional turbulent shear stress across the mixing layer is similar at any cross section. Verification of such behavior is provided by the measurements of Bradbury[31] for plane jets in most of the developed region.

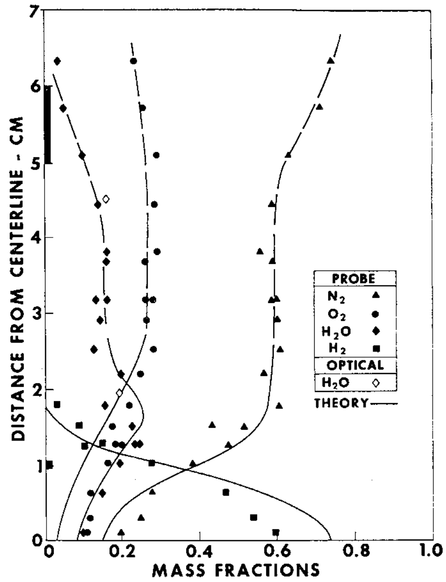


Fig. 14 Predicted concentration profiles at $x = 25.4$ cm compared with data—vitiated air condition.

V. Discussion of Results

Consistency Calculation

The total (integrated) hydrogen flow rate was computed at several axial stations for the vitiated nitrogen test conditions as an over-all check of the dependability of the species concentration, Pitot pressure, and temperature measurements. The calculated weight flows were approximately 27% lower than those determined from venturi pressure measurements. The primary cause of the deviations was traced to inaccuracies in the temperature measurements caused by the averaging which occurs over the 4.76-mm-diam probe inlet hole.

Comparison of Analytical Predictions with Data

The velocity ratio, u_a/u_i for the conditions tested was 0.749 suggesting that mixing was controlled exclusively by the preturbulence mechanism, i.e., $x_1 = 0$. Accordingly, the kinematic eddy viscosity model given by Eq. (8) with $m_1 = 0.6$ and $n_1 = 2.69$, was utilized with the mixing and combustion computer program to generate detailed flow properties for comparison with the measurements. Predicted species concentration profiles at $x = 25.4$ cm for the vitiated nitrogen condition are compared with data in Fig. 13. The trends of the data are reproduced accurately and, to a significant degree, good quantitative agreement is achieved. It is particularly notable that the observed bump in the water concentration profile is also generated by the analysis. The corresponding information for the vitiated air test condition in Fig. 14 shows that the use of the Eq. (8) mixing model leads to results which are predictive of the experimentally determined species concentration profiles. It is noted that the predicted centerline concentrations for the vitiated air condition are not as faithful to the data as those for the vitiated nitrogen condition. Nevertheless, the over-all agreement considering all the data is fully as satisfactory as that achieved for the nitrogen test condition. It follows, therefore, that mixing proceeds at essentially identical rates for the vitiated nitrogen and vitiated air test conditions notwithstanding the substantial differences in heat release rate and total heat evolved for these cases. A complete compilation of data and additional comparisons are given in Ref. 28.

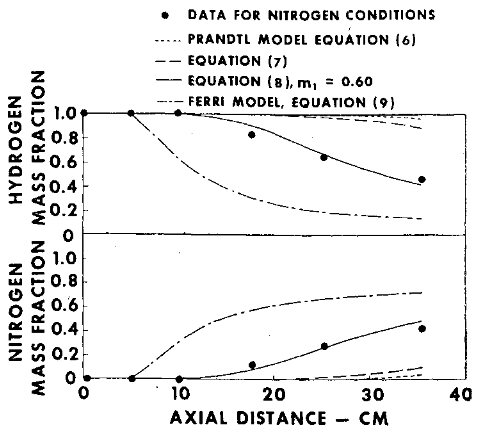


Fig. 15 Centerline concentration decay; comparison of data with different mixing models.

Evaluation of Kinematic Eddy Viscosity Models

The comparisons of data and theory presented have provided ample verification of the kinematic eddy viscosity model, Eq. (8), based on a preturbulence mechanism. However, it is also of interest to test other frequently used mixing models with the same data, Fig. 15. Prandtl's constant exchange coefficient of Eq. (6) predicts very low rates of mixing for the almost equal velocity jets studied. Inclusion of a density correction as per Eq. (7) increases the mixing only slightly. Another extension of Prandtl's ideas to include the effect of density variation through the mixing layer was proposed by Ferri.[32] This model has the form

$$\rho\epsilon = 0.025 r_{1/2}(\rho_a u_a - \rho_{\text{℄}} u_{\text{℄}}) \tag{9}$$

where $r_{1/2}$ is measured from one edge of the mixing layer to the position at which the local product of velocity and density, ρu, has an average value between the centerline value and the outer stream value, i.e.,

$$(\rho U)_{r1/2} = 0.5(\rho_{\text{℄}} u_{\text{℄}} + \rho_a u_a) \tag{10}$$

Application of Eq. (9) is seen to result in unrealistically high mixing rates.

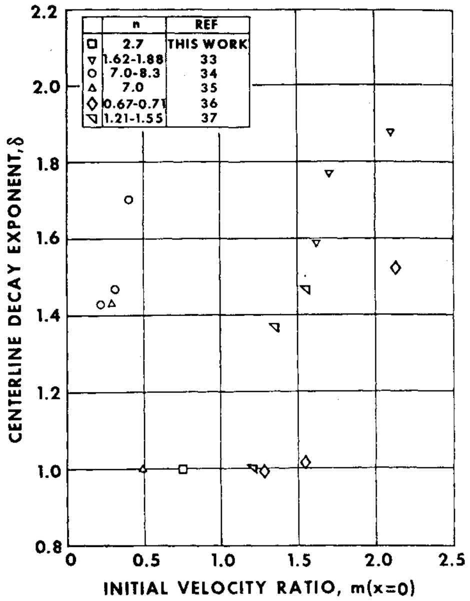
Comparison of Experimental Centerline Decay Rates

The variation of centerline concentration is generally presented in the form $M_{j,\text{℄}} = (x/x_0)^{-\delta}$, where x_0 is the core length. Various values of δ have been reported: Zakkay et al.[33] obtained $\delta = 2$; the data of Chriss[34] were correlated using $\delta = 1.70$; the present work yielded a value of 1.0 (see Fig. 9). The discrepancy which appears to exist between the various data can be resolved by constructing a plot of δ vs the initial jet velocity ratio, u_a/u_i, i.e., $m(x = 0)$. When this is done, Fig. 16, it is found that δ is unity over the range of velocity ratios where preturbulence dominates, independent of density ratio. Over the range of velocity ratios where jet (or wake) interactions are the dominant turbulence producing mechanism, δ is larger than unity and depends on both m and n. These findings are consistent with the jet spread data of Ref. 35.

VI. Conclusions

Detailed probe and optical measurements have been made in mixing and combusting free jets to obtain species concentrations, Pitot pressures and temperatures. On the basis of this data and its subsequent application to the verification of analytical mixing and chemistry models, it is concluded that:

1) The use of a kinematic eddy viscosity formulation based on a preturbulence mechanism in a constant pressure mixing and combustion analysis allows the accurate prediction of local conditions in freejet flowfields. The contribution to the turbulent transport from the jet shear interaction mechanism is not of importance, therefore, when the velocity ratios of jets exceed about 0.60. This finding is of consequence in all jet mixing problems, at least when considering the region far downstream from the nozzle exit plane.



	n	REF
□	2.7	THIS WORK
▽	1.62-1.88	33
○	7.0-8.3	34
△	7.0	35
◇	0.67-0.71	36
◁	1.21-1.55	37

Fig. 16 Experimental centerline decay exponents.

2) No major influence of chemical heat release on mixing rates exists at the conditions of the present investigation.

3) Temperatures and water concentrations measured spectrophotometrically using IR emission and absorption follow the same trends as comparable information obtained with probes, but only fair quantitative agreement is realized. Thus although the worth of spectrophotometric methods for the measurement of temperature and concentration data has been shown, this approach is still in a state of evolution. Clearly, optical techniques offer the only hope of acquiring local temperature information in flows having total temperatures in excess of 2000°K.

4) The ability to aspirate representative samples of gas from a reaction zone into a probe, effectively quench the sample and finally conduct an on-line gas chromatographic analysis of the sample has been demonstrated.

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